Exercise 38

In Exercises 29–40, test for symmetry with respect to each axis and to the origin.

$$y = \frac{x^5}{4 - x^2}$$

Solution

Replacing x with $-x$ changes the equation, so there's no symmetry with respect to the y-axis.

$$y = \frac{(-x)^5}{4 - (-x)^2} = \frac{-x^5}{4 - x^2} = -\frac{x^5}{4 - x^2}$$

Replacing y with $-y$ changes the equation, so there's no symmetry with respect to the x-axis.

$$-y = \frac{x^5}{4 - x^2} \quad \rightarrow \quad y = -\frac{x^5}{4 - x^2}$$

Replacing x with $-x$ and y with $-y$ does not change the equation, so there is symmetry with respect to the origin.

$$-y = \frac{(-x)^5}{4 - (-x)^2} \quad \rightarrow \quad -y = \frac{-x^5}{4 - x^2} \quad \rightarrow \quad y = \frac{x^5}{4 - x^2}$$